Adı Soyadı :
Numara :

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MAT 303 DİFERENSİYEL GEOMETRİ I FİNAL SINAVI SORULARI

SORU 1: $f: E^n \to IR$, $f(x_1, x_2, ..., x_n) = x_1x_2 + x_3^2 + x_n^3$ ve $P = (1,1,0,...,0) \in E^n$, $V = (1,1,...,1) \in IR^n$ ise $\vec{V}_P[f] = ?$

SORU 2: $[,]$, $\chi(E^n)$ de Lie operatörü olmak üzere $X = e^{x_2}\dfrac{\partial}{\partial x_1} - x_1x_2\dfrac{\partial}{\partial x_2} + x_3^2\dfrac{\partial}{\partial x_3}$ ve $Y = x_1x_3\dfrac{\partial}{\partial x_1} + \sin x_2\dfrac{\partial}{\partial x_2} + x_1^2\dfrac{\partial}{\partial x_3}$ vektör alanları ise $[X,Y] = ?$

SORU 3: Gradient fonksiyonunu tanımlayınız ve lineer olduğunu gösteriniz.

SORU 4: $F: E^3 \to E^3$, $F(x_1, x_2, x_3) = (e^{x_1} + 2x_2, 2\cos x_3, x_1x_2x_3)$ dönüşümü veriliyor. $P = \left(1,1,\dfrac{\pi}{2}\right)$ için $F_*|_P$ türev dönüşümünün $J(F,P)$ matrisini bulunuz.

SORU 5: E^n de regüler her eğrinin birim hızlı olacak şekilde bir koordinat komşuluğu vardır. İspatlayınız.

Not: Sorular eşit puanlı ve süre 90 dakikadır.

Başarılar
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= CEVAP ANAHTARI =

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$P = (1,1,0,...,0) \in E^n$, $V = (1,1,1,...,1) \in IR^n$ ve

$f(x_1, x_2, ..., x_n) = x_1x_2 + x_3^2 + x_n^3$ olmak üzere

$$\vec{V}_P[f] = \sum_{i=1}^{n} v_i \frac{\partial f}{\partial x_i}\Big|_P$$

$$= v_1\frac{\partial f}{\partial x_1}\Big|_P + v_2\frac{\partial f}{\partial x_2}\Big|_P + v_3\frac{\partial f}{\partial x_3}\Big|_P + \dots + v_n\frac{\partial f}{\partial x_n}\Big|_P$$

$$= 1.\ x_2\big|_P + 1.\ x_1\big|_P + 1.\ 2x_3\big|_P + 0 + 0 + \dots + 1.\ 3x_n^2\big|_P$$

$$= 1 + 1 + 0 + \dots + 0 = 2$$

bulunur.

(2) $[X,Y] = D_X Y - D_Y X$ biçimindeydi.

$D_X Y = \left(X(x_1x_3),\ X(\sin x_2),\ X(x_1^2) \right)$ dır.

$$X(x_1x_3) = \left(e^{x_2}\frac{\partial}{\partial x_1} - x_1x_2\frac{\partial}{\partial x_2} + x_3^2\frac{\partial}{\partial x_3} \right)(x_1x_3)$$

$$= e^{x_2}\frac{\partial(x_1x_3)}{\partial x_1} - x_1x_2\frac{\partial(x_1x_3)}{\partial x_2} + x_3^2\frac{\partial(x_1x_3)}{\partial x_3}$$

$$= x_3 e^{x_2} + x_1 x_3^2$$

$$X(\sin x_2) = \left(e^{x_2}\frac{\partial}{\partial x_1} - x_1x_2\frac{\partial}{\partial x_2} + x_3^2\frac{\partial}{\partial x_3} \right)(\sin x_2)$$

$$= e^{x_2}\frac{\partial(\sin x_2)}{\partial x_1} - x_1x_2\frac{\partial(\sin x_2)}{\partial x_2} + x_3^2\frac{\partial(\sin x_2)}{\partial x_3}$$

$$= -x_1x_2\cos x_2$$

$$X(x_1^2) = \left(e^{x_2}\frac{\partial}{\partial x_1} - x_1x_2\frac{\partial}{\partial x_2} + x_3^2\frac{\partial}{\partial x_3} \right)(x_1^2)$$

$$= e^{x_2}\frac{\partial(x_1^2)}{\partial x_1} - x_1x_2\frac{\partial(x_1^2)}{\partial x_2} + x_3^2\frac{\partial(x_1^2)}{\partial x_3}$$

$$= 2x_1 \cdot e^{x_2}$$

olup

$$D_X Y = \left(x_3 e^{x_2} + x_1x_3^2,\ -x_1x_2\cos x_2,\ 2x_1 e^{x_2} \right)$$

olur.

$D_Y X = \left(Y(e^{x_2}),\ Y(-x_1x_2),\ Y(x_3^2) \right)$ dır.

$$Y(e^{x_2}) = \left(x_1x_3\frac{\partial}{\partial x_1} + \sin x_2\frac{\partial}{\partial x_2} + x_1^2\frac{\partial}{\partial x_3} \right)(e^{x_2})$$

$$= x_1x_3\frac{\partial(e^{x_2})}{\partial x_1} + \sin x_2\frac{\partial(e^{x_2})}{\partial x_2} + x_1^2\frac{\partial(e^{x_2})}{\partial x_3}$$

$$= e^{x_2}.\sin x_2$$

$$Y(-x_1x_2)=\left(x_1x_3\frac{\partial}{\partial x_1}+\sin x_2\frac{\partial}{\partial x_2}+x_1^2\frac{\partial}{\partial x_3}\right)(-x_1x_2)$$

$$=x_1x_3\frac{\partial(-x_1x_2)}{\partial x_1}+\sin x_2\frac{\partial(-x_1x_2)}{\partial x_2}+x_1^2\frac{\partial(-x_1x_2)}{\partial x_3}$$

$$=-x_1x_2x_3-x_1\sin x_2$$

$$Y(x_3^2)=\left(x_1x_3\frac{\partial}{\partial x_1}+\sin x_2\frac{\partial}{\partial x_2}+x_1^2\frac{\partial}{\partial x_3}\right)(x_3^2)$$

$$=x_1x_3\frac{\partial(x_3^2)}{\partial x_1}+\sin x_2\frac{\partial(x_3^2)}{\partial x_2}+x_1^2\frac{\partial(x_3^2)}{\partial x_3}$$

$$=2x_1^2x_3$$

olup

$$D_YX=\left(e^{x_2}\cdot\sin x_2,\ -x_1x_2x_3-x_1\sin x_2,\ 2x_1^2x_3\right)$$

olur. Buradan

$$[X,Y]=D_XY-D_YX$$
$$=\left(x_3e^{x_2}+x_1x_3-e^{x_2}\sin x_2,\ -x_1x_2\cos x_2+x_1x_2x_3+x_1\sin x_2,\ 2x_1e^{x_2}-2x_1^2x_3\right)$$

elde edilir.

(3) Ders notlarında mevcuttur.

(4) $f_1(x_1,x_2,x_3)=e^{x_1}+2x_2$, $f_2(x_1,x_2,x_3)=2\cos x_3$ ve $f_3(x_1,x_2,x_3)=x_1x_2x_3$ olsun. Yine $P=(1,1,\frac{\pi}{2})=(p_1,p_2,p_3)$ alalım.

$$J(F,P)=\begin{bmatrix}\left.\frac{\partial f_1}{\partial x_1}\right|_P & \left.\frac{\partial f_1}{\partial x_2}\right|_P & \left.\frac{\partial f_1}{\partial x_3}\right|_P\\ \left.\frac{\partial f_2}{\partial x_1}\right|_P & \left.\frac{\partial f_2}{\partial x_2}\right|_P & \left.\frac{\partial f_2}{\partial x_3}\right|_P\\ \left.\frac{\partial f_3}{\partial x_1}\right|_P & \left.\frac{\partial f_3}{\partial x_2}\right|_P & \left.\frac{\partial f_3}{\partial x_3}\right|_P\end{bmatrix}=\begin{bmatrix}e^{x_1}(P) & 2(P) & 0(P)\\ 0(P) & 0(P) & -2\sin x_3(P)\\ x_2x_3(P) & x_1x_3(P) & x_1x_2(P)\end{bmatrix}$$

$$=\begin{bmatrix}e^{p_1} & 2 & 0\\ 0 & 0 & -2\sin(p_3)\\ p_2p_3 & p_1p_3 & p_1p_2\end{bmatrix}=\begin{bmatrix}e & 2 & 0\\ 0 & 0 & -2\\ \frac{\pi}{2} & \frac{\pi}{2} & 1\end{bmatrix}$$

(5) Ders notlarında mevcuttur.